Table 1 Bounds on manufacturing coefficient

k_1	Stepped thickness (rather than tapered)	0.1 . . 0.13
k_2	Dead joint mass penalty	0.15 . . 0.3
k_3	Standard thickness of webs, ribs, and other elements	0.1 . . 0.13
k_4	Joint fittings and joint defects	0.1 . . 0.15
k_5	Plus toleerances	0.04 . . 0.09
k_6	Manufacturing thicknesses	0.03 . . 0.05
k_7	Breakdown joint mass penalty	0.1 . . 0.2
	$k_{man} = 1 + k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7$	1.62 . . 2.05

Table 2 Sample calculations for current transports

Aircraft	μ, kg	m_s, actual	m_s, formula	Error, %
B-727-100	72,600	0.111	0.1109	− 0.02
B-747-100	322,000	0.122	0.1248	2.3
DC-9-30	49,000	0.106	0.0946	−10.8
DC-10-10	195,000	0.114	0.1048	−8.1
Dc-10-30	252,000	0.106	0.1109	4.6
A-300-B2	137,700	0.145	0.1337	− 7.8
C-5A	349,000	0.13	0.1344	3.4
C-130E	68,700	0.0772	0.0759	−1.7
Tu-154	90,000	0.102	0.1066	4.6
An-10	54,000	0.0981	0.1028	4.8
An-22	250,000	0.119	0.1226	9.5
An-24	21,000	0.1142	0.1196	4.7
Il-76T	171,000	0.121	0.1237	2.2

where Λ_m is the half-chord sweep of midboard wing; Λ_o is the half-chord sweep of outboard wing; m_w^* is the previously iterated or expert-estimated relative wing structure mass; m_{fu} is the relative fuel mass; $h_s = t_s\lambda_s/t_r$ is the thickness taper to sweep joint; $h_t = t_t\lambda_t/t_r$ is the thickness taper to wing tip; $\psi_s = h_s\lambda_s$ is the wing airfoil area taper to the sweep joint; t_s is the sweep joint relative thickness; t_t is the wing tip relative thickness; and $\psi_t = h_t\lambda_t$ is the wing airfoil area taper to wing tip. The formula cannot be used for a nontapered wing because division by zero will occur if $h_s = 1$ and $h_s = h_t$. If the wing has no sweep joint, it is suggested that

$$z_s = \frac{1}{2}; \quad \Lambda_m = \Lambda_o; \quad \lambda_s = 1 - \frac{1 - \lambda_t}{2 - 2z_f};$$

$$h_s = 1 - \frac{1 - h_t}{2 - 2z_f}; \quad \psi_s = 1 - \frac{1 - \psi_t}{2 - 2z_f} \tag{8}$$

Neither simplifications nor data for mass of existing wings have been applied, so the formula completely corresponds to the concept model.[5]

Applications

The newly derived formula has been used to study the transport aircraft in Table 2. The detailed spreadsheet calculations are presented in Ref. 5. Values of the actual stresses were estimated on the basis of engineering judgment. The accuracy of the formula is within [−10.8, +9.5]% and the root-mean-square error is 5.9%. This is sufficient for most preliminary design purposes. The formula can also be improved by using statistical data.

For comparison, a previously derived formula for twin fuselage aircraft[4] can be applied to a conventional aircraft. The results yield accuracy of [−13.1, +11.7]% with root-mean-square error of 7.0%. This is not as accurate as the present formula.

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Whirl-Flutter Stability of a Pusher Configuration in Nonuniform Flow

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Introduction

IN the new aircraft configurations with pusher power plants located at the rear fuselage, the whirl-flutter stability characteristics of the engine-propeller installation may be affected by the nonuniform freeflow induced by the aircraft components positioned upstream in the flowfield. Assuming that the flowfield may still be considered stationary, the effect of the aforementioned interference is investigated. Under this assumpton, the propeller aerodynamics may be adjusted to become a periodic function of time and the aeroelastic system stability may be studied using any theory developed to analyze periodic systems. In the present work, Floquet-Liapunov's theorem[1] could be applied efficiently to evaluate the stability characteristics of an actual configuration with many interference elements.

Basic Formulation

The equation of motion of the aeroelastic system is cast in the state vector form

$$\dot{y} = A(t)y \tag{1}$$

where $A(t)$ is the matrix of influence coefficients containing the information about both the dynamics and the aerodynamics of the system. For a given set of initial conditions, the solution of this system of ordinary differential equations may be written as

$$y(t) = \Phi(t, t_0)y(t_0) \tag{2}$$

where $\Phi(t, t_0)$ is recognized to be the transition matrix relating the state vector y at the instant t to its initial value at t_0. Floquet's theorem states that in order to completely evaluate the stability characteristics of a periodic system, it will suffice

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to study the nature of the eigenvalues of the transition matrix calculated over a single period. Among the many applications based on this useful theorem, the stability of rotors has been extensively investigated.[2,3] In particular, Nitzsche used the same formalism that is followed in this work to analyze the stability of vertical axis wind turbines subject to a nonzero wind speed.[4] Here, the formulation of the aeroelastic equations is based on Houbolt and Reed's classical paper on the whirl-flutter problem of a two-degree-of-freedom pitch-yaw system.[5] It is enhanced with the necessary inclusion of both the lateral and the vertical degrees of freedom of the engine suspension system, yielding a four-degree-of-freedom problem. This approximation is valid if the back-up structure normal modes are shown to play no important role on the system stability.[6,7]

The effect of the aerodynamic interference induced by the aircraft components located upstream the propeller disk (nacelle, pylon, wing, and fuselage), is introduced by two periodic functions of time, respectively associated with the blade local angle-of-attack $\alpha_0(t)$, and the resultant wind velocity $\beta_0(t)$. The introduction of such functions may be understood on physical grounds. For an observer sitting on the blade rotating frame, the aerodynamic perturbation associated with the aforementioned aircraft components generates a cyclic change in the local angle of attack (with the propeller spinning frequency Ω). This angle-of-attack variation is isolated by decomposing the perturbation velocity vector $\boldsymbol{V}_{p0} = \boldsymbol{V}_t - \boldsymbol{V}$ in two directions—normal to the propeller disk and tangential to the blade circular path

$$\boldsymbol{V}_{p0} = \dot{u}_c(\boldsymbol{V}/|\boldsymbol{V}|) + \dot{s}_c(\boldsymbol{\Omega r}/|\boldsymbol{\Omega r}|) \tag{3}$$

where $\boldsymbol{V}$ and $\boldsymbol{\Omega r}$ are, respectively, the freeflow and the blade-section local tangential velocity vectors, and $\boldsymbol{V}_t$ is the total velocity vector associated with the nonuniform flow computed at the propeller disk. Hence

$$\alpha_0 = (-\Omega r/U^2)\dot{u}_c + (V/U^2)\dot{s}_c \tag{4}$$

where $U^2 = V^2 + \Omega^2 r^2$ is the blade-section local resultant velocity assuming the uniform flow conditions. Therefore, α_0 gives the change in the direction of the blade-section total resultant-velocity vector $\boldsymbol{U}_t$ due to the nonuniform flow that reaches the propeller, whereas

$$\beta_0 = 1 + 2[(\Omega r/U^2)\dot{s}_c + (V/U^2)\dot{u}_c] \tag{5}$$

expresses the corresponding change in the magnitude of $\boldsymbol{U}_t$, where

$$U_t^2 = U^2\beta_0 \tag{6}$$

Equations 4 and 5 are developed in Refs. 8 and 9. For the uniform flow, $\alpha_0(t) \equiv 0$ and $\beta_0(t) \equiv 1$. This approach allows the introduction of the periodic nature of the system by a simple modification in the propeller aerodynamic coefficients developed for the classical analysis. A numerical integration is performed to compute the aerodynamic coefficients over one complete cycle of the dynamic system. Calling $\boldsymbol{F}_{aed}$ the vector collecting the aerodynamic loads, one has

$$\boldsymbol{F}_{aed} = \boldsymbol{K}_{aed}(\alpha_0, \beta_0; t)\boldsymbol{y} + \boldsymbol{C}_{aed}(\alpha_0, \beta_0; t)\dot{\boldsymbol{y}} \tag{7}$$

where $\alpha_0(t)$ and $\beta_0(t)$ are evaluated for the particular flowfield and $\boldsymbol{K}_{aed}(\alpha_0, \beta_0; t)$ and $\boldsymbol{C}_{aed}(\alpha_0, \beta_0; t)$ are, respectively, the aerodynamic stiffness and damping matrices.

Aerodynamics—Nonuniform Flowfield

The propeller aerodynamics are based on the blade element theory. The oscillatory airloads are supposed to be generated by two independent sources; namely 1) a change on the blade local angle of attack with respect to the undisturbed airflow due to the propeller dynamics; and 2) a change on the blade angle of attack due to a perturbation in the uniform flowfield.

Neglecting the drag contribution, for an element of length dr located at a distance r along the blade, the elementary lift total perturbation reads

$$dL = \tfrac{1}{2}\rho c U^2 \beta_0(t) c_{L\alpha}[\alpha_0(t) + \alpha_1 + \alpha_2]\, dr \tag{8}$$

where ρ is the air density, c is the local chord, and c_{L_α} is the local blade lift-curve slope; α_1 and α_2 are perturbations in the blade local angle of attack due to 1) the pitch-yaw angular displacement of the propeller disk with respect to the undisturbed wind velocity; and 2) the velocity vector induced by the motion of the propeller disk with respect to the undisturbed wind velocity. The latter perturbation angles are well known in the literature and were derived in a previous work.[6]

The airload is first computed for a single blade at an angular position defined by Ωt. For a propeller with B blades the azimuth angle ξ_i, defining the position of the ith blade, is given by

$$\xi_i = \Omega t + 2\pi/B(i - 1) \qquad i = 1, 2, \ldots, B \tag{9}$$

Hence, the characteristic period of the system is $T = 2\pi/(\Omega B)$. To arrive to the total airload generated at the propeller hub it is necessary both to integrate the elementary contributions along the blade span and to sum for the B blades. The result is then identified with Eq. 7, yielding after linearizations with respect to either α_0 or β_0 closed mathematical expressions for $\boldsymbol{K}_{aed}$ and $\boldsymbol{C}_{aed}$.[8,9]

Numerical Solution

As discussed in the introduction of this work, Floquet's theorem is used to investigate the stability characteristics of the periodic system which results from the present formulation. Hsu's method[10] is employed to compute the transition matrix in the time interval representing a complete period, $\boldsymbol{\Phi}(T,0)$. Standard computer library routines are used as a final step to extract Λ_j, the complex eigenvalues of $\boldsymbol{\Phi}(T,0)$. The latter have their magnitudes and phase angles related to the modal damping of the corresponding aeroelastic modes

$$Re(\lambda_j) = 1/T[\ln(\Lambda_j) + i \arg(\Lambda_j)] \tag{10}$$

where $Re(\lambda_j)$ stands for the real part of the jth complex eigenfrequency.

Whirl-Flutter Analysis

The configuration shown in Fig. 1 is then considered. In this case, the flowfield at the plane of the propeller is numerically computed by the panel method. The perturbation angle $\alpha_0(t)$ is obtained with a 1558-panel idealization of the aircraft. For the numerical integration of the propeller aerodynamic coefficients, a discretization of 1800 points over the propeller disk was taken (20 radial times 90 angular positions). Figure 2 presents the result for which the stability boundaries for two different vortex sheets, obtained by assuming both zero and nonzero circulation for the pylon, is investigated. The classical solution, obtained with the uniform flow assumption, is represented by the solid line. The pylon nonzero-lift vortex sheet, indicated by the number 2 in the same figure, caused a reduction in the stability margin, whereas the pylon zero-lift vortex sheet (No. 1) did the opposite. Several similar studies demonstrate that the stability is strongly dependent upon the trailing-edge departure angle assumed for the vortex sheet associated to the pylon circulation. Because this angle may be associated with the total lift generated by the pylon, it follows that the stability is a function of the airplane attitude angle. Therefore, different high-speed flying conditions must be checked to assure a reliable whirl-flutter analysis.

Figure 3 presents another interesting result. The whirl flutter stability of the engine-propeller installation immersed in a perturbed airflow is not only dependent upon the distance d, defining the distance between the engine-propeller center-of-mass and the suspension elastic center, as indicated by the classical analysis,[5] but also upon the nature of the critical mode. According to Fig. 3, in the range of values of d for which the yaw mode is critical, as d increases, the system stability reduces itself, regardless of the pylon circulation condition. This trend reverses itself for larger positive values of d, for which the vertical mode becomes critical. Furthermore, assuming that the vortex sheet No. 2 is representative of the flying condition, the global effect of the perturbations on the whirl-flutter stability is slightly detrimental in the case of $d = -0.3$ m, highly detrimental in the case of $d = 0.02$ m, and again slightly detrimental for $d = 0.3$ m. However, if the vortex sheet No. 1 is adopted, there is an improvement on the stability limits, especially for $d = 0.3$ m or $d = -0.3$ m, even though a slight detriment is noticed for small positive values of d.

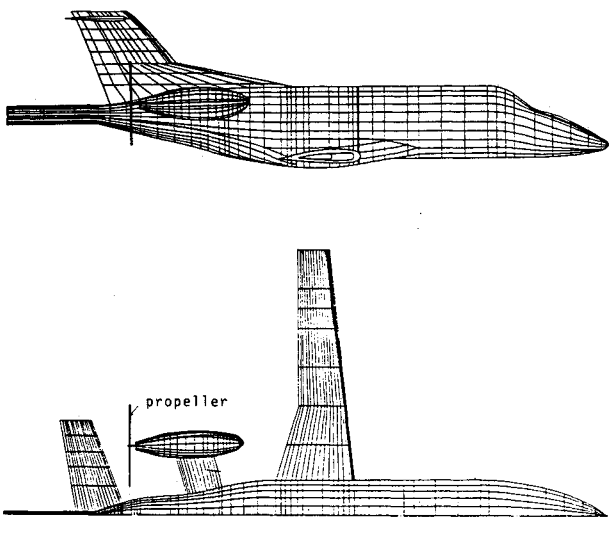

Fig. 1 Aircraft panel idealization.

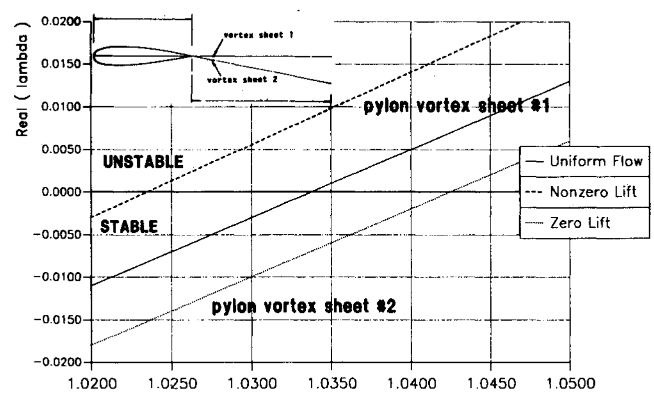

Fig. 2 Whirl-flutter stability boundaries for two different pylon circulations ($d = -0.05$ m); $J = V/\Omega R$, where R is the propeller radius.

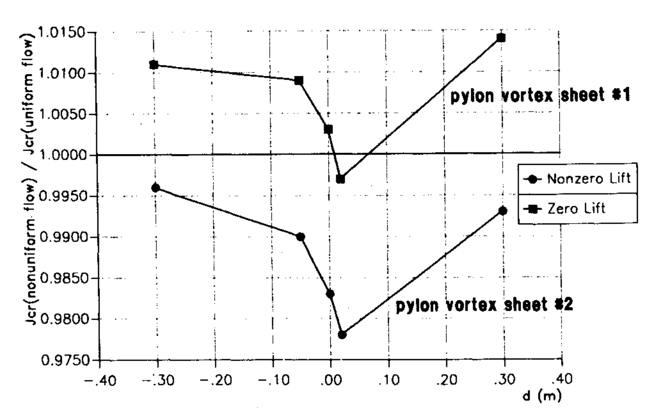

Fig. 3 Relative effect of the flow perturbations on the whirl-flutter stability for the pylon circulation conditions defined in Fig. 2; J_{cr} is the value of J associated with the whirl-flutter onset.

Conclusions

In the present work an answer to the question concerned with the influence of aerodynamic interference elements on the whirl-flutter stability of advanced turboprops with pusher propellers was sought. A simple mathematical model that allows the use of theories developed for periodic systems was constructed and some of the most important aspects of the problem could be quantified.

In the chosen example, the influence of the flow perturbations on the whirl-flutter stability was never larger than 2.2%, indicating that although significant values of flow deviation were observed in the idealization,[8,9] the overall effect of the aerodynamic interference was such that it could be ignored in a reliable whirl-flutter analysis. However, this conclusion should not be regarded as general for all aircraft employing the pusher propeller configuration. Further investigation may demonstrate that the number of blades (in particular whether it is even or odd), and the distance between the propeller and the pylon—the suction effect quantified by the function $\beta_0(t)$—are of higher importance.

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